Determining the Stratification of Exchange Flows in Sea Straits

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LONG-TERM GOAL

Our long-term goal is to contribute to our understanding of the dynamics of the exchange flow and the stratification in the Bab al Mandab and its effect on the general circulation and stratification of the neighboring Red Sea and Gulf of Aden.

OBJECTIVES

We want to determine whether exchange flow through the Bab al Mandab is hydraulically controlled and what the implications are for the stratification in the strait and surrounding waters. Consideration of these questions will involve tides and mixing. We are particularly interested in critical layer mixing near points of hydraulic control and its consequences for the stratification. We further seek to better understand the processes by which the exchange flow is coupled to the circulation in the Red Sea. Such processes undoubtedly depend of rotation. A problem of particular interest is determining how the upper layer approaches the Bab al Mandab from the Red Sea (and how the lower layer enters the Red Sea.)

APPROACH

Our approach combines data analysis with analytical and numerical modeling. Specific modeling efforts are motivated by questions arising from consideration of the data and, in some cases, from consideration of fundamental mysteries surrounding continuously stratified exchange flow. The data comes from past and present field work in the Bab al Mandab by Profs. Bill Johns (RSMAS) and Steve Murray (LSU/ONR). Analytical tools include various advanced techniques for finding the propagation speeds of long internal gravity waves for stratified shear flows in channels with nonuniform cross-sections. The numerical models include a 2-D code for calculating time-dependent, nonhydrostatic flow over an obstacle and a 3-D code for calculating two-layer flows in rotating channel/basin sys-tems. Since a number of fundamental questions involving the interplay between hydraulic control and mixing remain unanswered, our calculation generally involve idealized settings rather than exact replication of field conditions.

WORK COMPLETED

In order to better evaluate the hydraulic criticality of the flow at the narrows and sill, it was necessary to develop an extended Taylor-Goldstein equation (Pratt *et al*., 2000). The new equation allows for the calculation of the phase speeds of long internal waves for a stratified shear flow in a channel of arbitrary cross-section. Theorems that proceed from this equation are potentially valuable in determining the conditions under which critical levels, instability, and consequential mixing can occur. Some of our work over the past year has been devoted to establishing these theorems. We have collaborated with Prof. Chris Jones Mr. Jian Deng (both of Brown) and Prof. Lou Howard (FSU) in this effort.

Results from the previous year, as reported in Pratt *et al*. (1999 and 2000) indicated that critical conditions tend to occur at the sill (and not the narrows) and then only during certain phases of tides or subtidal disturbances. In order to evaluate the critical level behavior and mixing that would, in principle, result from the intermittent appearance of critical flow, we have developed and are testing a nonhydrostatic model for 2-D stratified flow. An example of a test simulation appears in Figure 1 which shows the stratification of an exchange flow over one tidal cycle.

Concerning our efforts to understand the effects of rotation and coupling with the Red Sea, Helfrich has developed a two-layer model that can accommodate special features including hydraulic jumps and bores that arise in hydraulically driven flows. Further, the model can accommodate strait and basin geometries with general topography. Pratt has been developing an analytical two-layer, rotating hydraulic model and we are getting ready to test this theory against the numerical code.

RESULTS

Our investigations of the extended Taylor-Goldstein equation have resulted in the formulation of necessary conditions for instability and bounds on the growth rate and phase speeds. These bounds are very useful in interpreting the observations due to Johns and Murray. In particular, we can better anticipate the conditions under which critical flow occurs and critical level mixing is possible.

Since most of our other efforts have been directed at model development, we have few new results to report on. However, Pratt's work on the two-layer, rotating hydraulic model has yielded a transport relation making it possible to obtain exchange transport at a control section with a single measurement of interface height. It will be interesting to compare this to transports obtained by Johns and Murray from there past and ongoing work (*e.g.*, Murray and Johns, 1997).

TRANSITIONS

As part of the establishment of stability conditions based on the extended Taylor–Goldstein equation, we have developed a new mathematical approach to proving stability. It is not clear yet how general this approach is, but it may prove useful in other stability problems. The two numerical codes (the rotating, two-layer, shock-capturing code and the 2-D nonhydrostatic code) are completed are will be available to the community at large after they are tested.
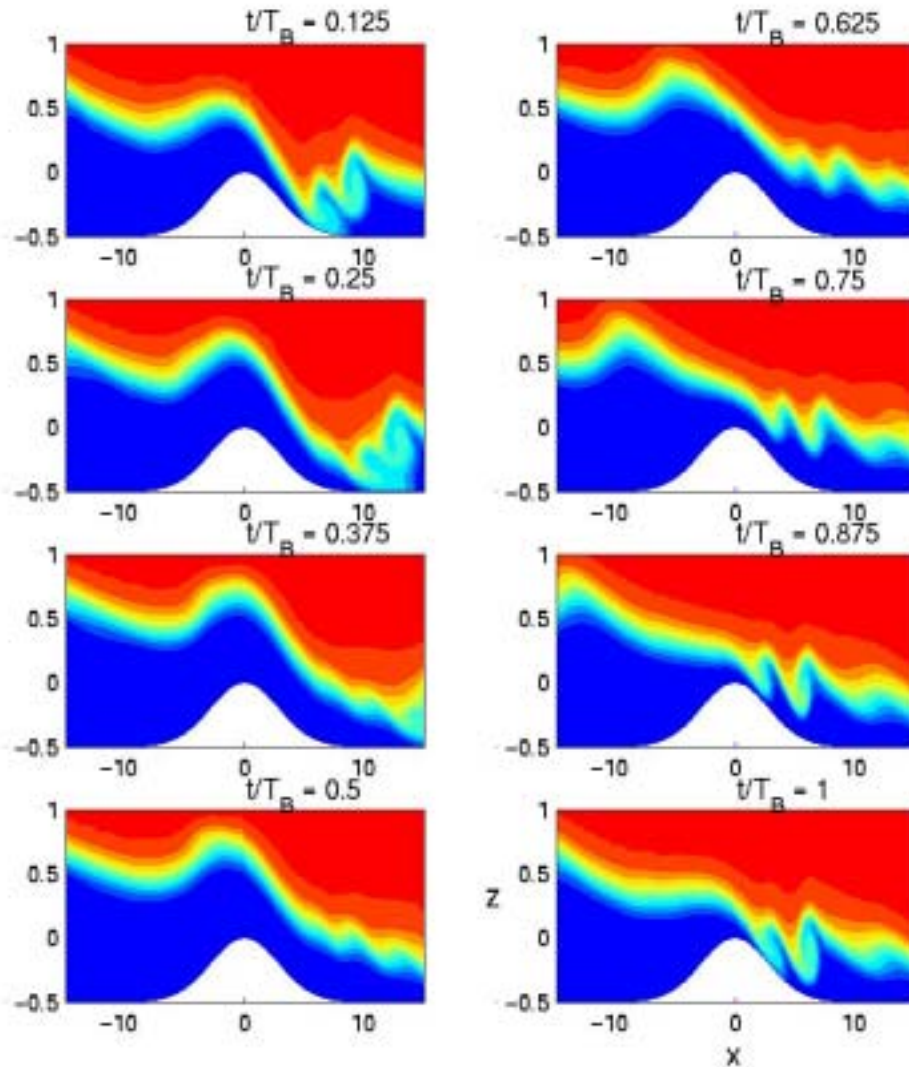
Figure 1. Density sections showing the evolution of a continuously stratified exchange flow over a single tidal cycle based on a nonhydrostatic model.

RELATED PROJECTS

Drs. Bower and Fratantoni (of Woods Hole) along with Drs. Johns and Murray have recently completed a new field study of the Gulf of Aden outflow with some data collected inside the Bab al Mandab. We are eager to find out what new insights are provided relative to our interests.

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